Absorbing state phase transitions with a non-accessible vacuum

Omar Al Hammal,[1] Juan A. Bonachela,[1] and Miguel A. Muñoz[1]

[1] *Instituto de Física Teórica y Computacional Carlos I and Departamento de Electromagnetismo y Física de la Materia, Facultad de Ciencias, Universidad de Granada, 18071 Granada, Spain*

(Dated: February 6, 2008)

We analyze from the renormalization group perspective a universality class of reaction-diffusion systems with absorbing states. In this class, models where the vacuum state is not accessible are represented as the set of reactions $2A \to A$ together with creation processes of the form $A \to nA$ with $n \geq 2$. This class includes the (exactly solvable in one-dimension) *reversible* model $2A \leftrightarrow A$ as a particular example, as well as many other *non-reversible* sets of reactions, proving that reversibility is not the main feature of this class as previously thought. By using field theoretical techniques we show that the critical point appears at zero creation-rate (in accordance with known results for the reversible case) and it is controlled by the well known pair-coagulation renormalization group fixed point, with non-trivial exactly computable critical exponents in any dimension. Finally, we report on Monte-Carlo simulations, confirming the field theoretical predictions in one and two dimensions for various reversible and non-reversible sets of reactions.

PACS numbers: 02.50.-r,05.10.-a 05.10.Cc 11.10.Hi 64.60.Ht

INTRODUCTION

In a recent paper Elgart and Kamenev [1] have proposed a classification of absorbing state phase transitions, a subject that has been one of the central pillars of non-equilibrium statistical mechanics over the last decade [2, 3]. The strategy they follow is elegant and powerful. The main idea is to (i) write down using standard techniques the generating functional (or, equivalently, the effective Hamiltonian) for a given reaction-diffusion system; (ii) inspect the phase space in saddle-point approximation paying special attention to the "zero-energy" manifolds which determine the topological properties; (iii) detect possible structural changes in the phase portrait: the birthmark of phase transitions, and (iv) classify them according to basic topological properties. This procedure is a natural extension to non-equilibrium problems of the rearrangement of thermodynamic-potential minima occurring at equilibrium phase transitions. Hence, it allows for a categorization of universality classes attending to symmetry principles, conservation laws, and few other relevant ingredients, which determine the phase-space topology and its possible structural changes. Establishing the limits of validity of the saddle-point approximation within this context and developing systematic improvements to it remain as fundamental open problems.

Using this strategy, Elgart and Kamenev report on 5 non-trivial universality classes with absorbing states, occurring in one-dimensional systems with just one type of particle [4, 5]. The first 4 ones are: (i) *directed percolation* (DP) characterizing generic systems with an absorbing phase transition and without extra symmetries, conservation laws, quenched disorder, nor long-range interactions [3, 6], (ii) the usually called *parity conserved* (PC) [7] also known as DP2 or generalized voter class [8] which includes two symmetric absorbing states, (iii) the very elusive *pair-contact-process-with diffusion* (PCPD) class in which all reactions involve pairs of particles [9, 10], and (iv) the *triplet-contact-process-with diffusion* (TCPD) in which reactions involve triplets of particles [11].

In this paper we focus on the fifth class in [1]. It describes the reversible reactions $A \to 2A$ and $2A \to A$ occurring at rates μ and σ respectively. This model was solved exactly in one dimension more than twenty years ago in a seminal paper by Burschka, Doering, and ben-Avraham [12] by employing the *empty interval method* [13]. Finite-size properties, scaling functions, and critical exponents have also been exactly computed for this reversible model and for variations of it [12] in one dimension. Note that except for the absence of one-particle spontaneous annihilation, $A \to 0$, this set of reactions coincides with the contact process [2] a well-known model in the robust DP class [3]. It is, therefore, interesting to elucidate which is the main relevant difference in the renormalization group sense, giving rise to a non-trivial non-DP type of scaling. From considerations in [1] it seems that the fact that the reactions are reversible plays such a relevant role, but as we will illustrate, *reversibility is a sufficient, but not a necessary, condition.*

From the field theoretical point of view, Cardy and Täuber had obtained in their seminal article [14] a one-loop calculation of critical exponents for the closely related set of reactions $2A \leftrightarrow 0$, while in a recent paper Jack, Mayer, and Sollich have shown that such one-loop results are also valid for $2A \leftrightarrow A$ and have to be exact owing to the existence of *detailed balance* for reversible reactions [15]. Therefore, two or more loop corrections should cancel out, even if this is not explicitely shown in [15]. In any case, the main results are that the critical point is located at $\mu_c = 0$ (any non-vanishing branching rate leads

to sustained activity) and the order-parameter critical exponent is $\beta = 1$. The long-time long-distance properties turn out to be controlled by the well-known "pure" pair-coagulation ($2A \to A$) RG fixed point [14, 15, 16, 17] and all exponents can be computed in any dimension.

In this paper, we perform a full diagrammatic expansion of various reaction-diffusion models extending previous analyses to all orders in perturbation theory. First, we recover the previously known results for the reversible model $2A \leftrightarrow A$. Afterward, using the intuition developed from the previous full diagrammatic analysis we construct different sets of *non-reversible* reactions, and argue that they belong to this same universality class. Its key ingredient turns out to be the absence of an accessible vacuum state, i.e. there is no reaction $mA \to 0$ but just pair-coagulation, combined with creation reactions of the form $A \to nA$. The *reversible reaction*, $n = 2$ discussed in [1] and [15] is just a representative of this broader class: reversibility (which tantamount to the detailed-balance condition in [15]) is a sufficient but not a necessary requirement.

Let us remark that reactions as $2A \leftrightarrow 0$ and its non-reversible extensions $2A \to 0$, $0 \to 3A,\ 4A, ...$ can also be argued to belong to this same class. In these cases, the vacuum state is accessible, but it is not stable, so they are not genuine absorbing state models.

To verify the field-theoretical predictions we perform Monte-Carlo simulations for various non-reversible sets of reactions, implemented with and without hard-core exclusion ("fermionic" or "bosonic", respectively) in one and two dimensions. All critical exponents, are in perfect agreement with the RG predictions, confirming the existence of a robust universality class, broader than thought before.

Before proceeding, we should underline that while many of the results contained in this paper are already known (some from exact solutions of the reversible model in one dimension [12] and some from similar perturbative calculations combined with symmetry arguments [1, 14, 15, 16, 17]), a systematic presentation of them, focusing the attention on universality aspects is, to the best of our knowledge, lacking in the literature. This paper aims at filling this empty space and at providing a comprehensive picture of this universality class, extending it to non-reversible reactions without an accessible vacuum state.

FIELD THEORY ANALYSIS OF $2A \leftrightarrow A$

The techniques employed in this section are standard and we refer the reader to [18, 19] and more specifically to [14, 16, 17] for more detailed calculations and/or pedagogical presentations.

Let us apply the Doi-Peliti formalism [14, 17, 18] (see also [20, 21]) to the reversible set of reactions $A \to 2A$ and $2A \to A$ occurring at rates μ and σ respectively. They can be cast into a generating functional whose associated (bosonic) action is

$$\begin{aligned} \mathcal{S}[\phi,\pi] &= \int dt \int d^dx [\pi(\partial_t\phi - D\nabla^2\phi) - H[\phi,\pi]], \quad with \\ H[\phi,\pi] &= (\pi^2 - \pi)(\mu\phi - \sigma\phi^2), \end{aligned} \tag{1}$$

where $\phi(\mathbf{x},t)$ and $\pi(\mathbf{x},t)$ are the density and the response fields respectively (some spatial and time dependences have been omitted for simplicity). For a general process $kA \to jA$ with k and j integer numbers, the associated effective Hamiltonian in this formalism includes a factor $[\pi^j - \pi^k]\phi^k$, which *is proportional to* $[\pi^2 - \pi]$ *if and only if the absorbing state is not accessible, i.e.* $j > 0$ *and* $k > 0$.

For readers with more intuition in terms of stochastic equations, an associated Langevin equation can be easily derived:

$$\partial_t\phi(\mathbf{x},t) = D\nabla^2\phi + \mu\phi - \sigma\phi^2 + \sqrt{\mu\phi - \sigma\phi^2}\eta(\mathbf{x},t) \tag{2}$$

where $\eta(\mathbf{x},t)$ is a Gaussian white noise. Let us emphasize the similarity between Eq.(2) and the Langevin equation for the DP class [3, 6]. Despite of this alikeness, Eq.(2) is not free from interpretation difficulties as the density field is not a real-valued one, but develops an imaginary part [21]. For this reason we avoid using it and center the forthcoming discussion on Eq.(1).

Owing to the fact that the effective Hamiltonian in Eq.(1), $H[\phi,\pi]$, is proportional to $(\pi^2 - \pi)$, $\pi = 0$ and $\pi = 1$ are zero-energy manifolds. The existence of these two constant-π solutions is, according to [1], at the basis of the non-DP behavior of this model. Indeed, the four zero-energy solutions: $\pi = 0$, $\pi = 1$, $\phi = 0$, and $\phi = \mu/\sigma$, define a rectangular geometry in the phase portrait (see Fig. 1b and [1]), which should be compared with the standard, triangular, DP topology, for which only one constant-π solution exists (see Fig. 1a and [1]) as we illustrate now.

It is worth noticing that the common factor $(\pi^2 - \pi)$ in Eq.(1), arising from the fact that the absorbing state is not accessible, can be interpreted as a *subtle symmetry* between all the coefficients of (noise) terms proportional to π^2 and their corresponding (deterministic) ones proportional to $-\pi$. Indeed, it is closely related to the detailed-balance symmetry discussed in [15]. If an additional reaction $A \to 0$ occurring at rate λ is switched on, a term $\lambda\phi(1 - \pi)$ has to be added to the Hamiltonian. In such a case, $(\pi^2 - \pi)$ is not a common factor, the subtle symmetry is broken and $\pi = 0$ is not a zero-energy solution anymore. This leads to the triangular topology for zero-energy manifolds in the phase space (Fig 1a) and, hence, to DP-scaling. Something similar occurs by switching on any other reaction as $mA \to 0$, with $m \geq 2$, converting the vacuum into an accessible state.

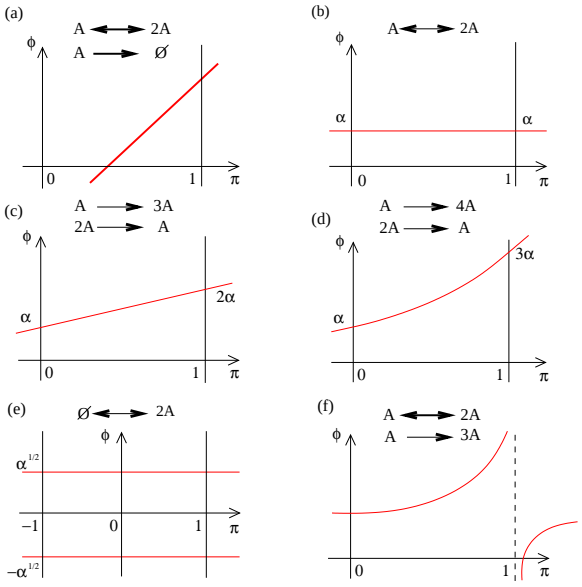
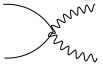


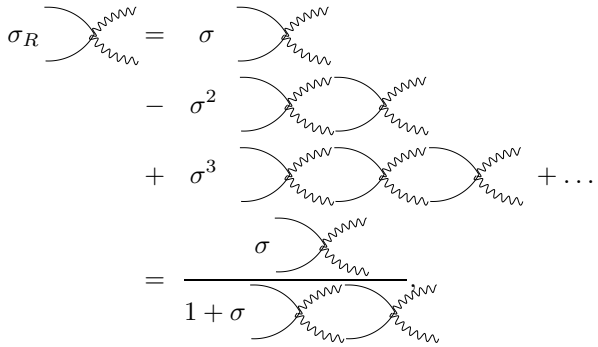
FIG. 1: Schematic zero-energy manifolds (bold lines) for different reactions. The line marked in red (non-trivial manifold, depending on the control parameter μ) moves downward upon approaching the critical point in all cases: $\alpha = \mu/\sigma$. While directed percolation is characterized by a triangular structure as in (a), models without an accessible vacuum state have a different form, being rectangular (b), trapezoidal (c), or more complicated geometries (d), for different reactions. For reversible models with an unstable vacuum, $\phi = 0$ is not a zero-energy manifold (e). The structures in (c) and (d) are not robust under RG flow, but evolve to non-closed topologies as the one in figure (f).

Let us present a different argument leading to the same conclusion. From standard naïve power counting and relevance arguments one could be tempted to conclude that this problem is in the DP class, and that the critical dimension is $d_c = 4$. Indeed, as said before, the leading (lowest order) terms in both the deterministic part and the noise are identical to their analogous ones in the DP field theory [3, 6], for which $d_c = 4$. The only way out of this naïve (and wrong) conclusion is that, at the critical point where the linear-deterministic term coefficient vanishes, the coefficient of the leading DP-like noise term $\pi^2\phi$ also vanishes owing to the abovementioned subtle symmetry. This opens the door for higher order noise terms to control the (non-DP) scaling. Indeed, a proper power counting analysis reveals that, as the interaction Hamiltonian is proportional to $(\pi^2 - \pi)$, π has to be dimensionless, which leads to $[\phi] = \Lambda^d$ (where Λ has dimensions of momentum) to ensure a dimensionless action, and consequently to $[\mu] = \Lambda^2$ and $[\sigma] = \Lambda^{2-d}$. Therefore, the theory upper critical dimension is $d_c = 2$ [1, 14, 15].

The existence of the common factor $(\pi^2 - \pi)$ in Eq.(1) implies that the μ-dependent non-trivial manifold and the trivial one, $\phi = 0$, merge at the critical point rather than intersecting in just a point as in DP. This is the key reason for the models without an accessible vacuum to exhibit a different type of scaling. To substantiate this assertion we need to prove that the previous bare-action symmetry, or associated topological structure, survives to the inclusion of fluctuations (i.e. it remains valid beyond mean-field [22]).

The basic elements for a complete perturbative expansion at a diagrammatic level are: the usual DP-noise vertex ($\pi^2\phi$) [6, 14, 17], the pair-coagulation ones ($\pi\phi^2$ and $\pi^2\phi^2$), as well as the propagator $(-i\omega + D\kappa^2 + \mu)^{-1}$ [14, 16, 17]. Diagrammatically, we represent response fields by wavy lines and the density fields by straight ones. For instance, the pair-coagulation noise vertex is depicted as σ^2  and analogous figures, with different numbers of straight and/or wavy lines are employed for the other vertices and the propagator.

In order to perform a sound perturbative expansion to all orders in perturbation theory, we choose to write separately diagrams with and without corrections proportional to μ. The second group includes only diagrams with vertices proportional to σ (i.e. expansions of Z-functions in powers of σ). Simple inspection reveals that such diagrammatic corrections are those of the pair-coagulation process, a theory well-known to be *super-renormalizable*, i.e., all these diagrams can be computed and resummed to all orders. Indeed, the only possible diagrammatic corrections to the pair-coagulation vertices, proportional to σ, have the typical "bubble" structure, leading to a geometric series



where σ_R is the renormalized (or "dressed") coagulation coefficient. Omitting external legs in Eq.(3), $\sigma_R = \frac{\sigma}{1+\sigma I} = \sigma(1 - \sigma\Sigma)$ with $\Sigma = \frac{I}{1+\sigma I}$, where I denotes the one-loop diagram evaluated at zero external frequency and arbitrary momentum scale Λ [14, 18, 19]:

$$I = \frac{1}{(2\pi)^{(d+1)}} \int d^d\kappa \; d\omega \; \frac{1}{i\omega + D\kappa^2} \; \frac{1}{-i\omega + D\kappa^2} \propto \frac{\Lambda^{-\epsilon}}{\epsilon} \tag{3}$$

with $\epsilon = 2 - d$. Similar expressions are obtained for all the renormalized coefficients by just changing the leftmost and/or the rightmost vertex of the series in Eq.(3). If these were the only corrections, (i.e. if the RG fixed point was at $\mu_c = 0$ so that diagrams including corrections proportional to μ would not give any non-vanishing

contribution) then the renormalized parameters would be:

$$\mu_R = Z_\mu \mu = \mu(1 - \sigma\Sigma)$$
$$\sigma_R = Z_\sigma \sigma = \sigma(1 - \sigma\Sigma). \quad (4)$$

for the two coefficients proportional to μ and the two proportional to σ respectively, showing that the subtle symmetry is not broken. The corresponding flow equations would be

$$\partial_l \mu_R = \mu(2 - \sigma\partial_l\Sigma)$$
$$\partial_l \sigma_R = \sigma(\epsilon - \sigma\partial_l\Sigma) \quad (5)$$

where ∂_l stands for the logarithmic derivative with respect to the momentum scale at which integral are evaluated [14, 17, 18, 19]. For $\epsilon < 0$, i.e. $d > 2$, the trivial (mean field) solution $\sigma = \mu = 0$ is infrared stable, while for $\epsilon > 0$, the only infrared stable fixed point is $\sigma* = \epsilon/(\partial_l\Sigma)$ with $\mu = 0$. Plugging this into the first equation in (5), we obtain the anomalous scaling dimension of μ_R, $[\mu_R] = 2 - \epsilon = d$, which coincides with the one-loop result obtained in [1, 14] (see also [15]).

The change in the scaling dimension of the "mass" term, from its naïve value $[\mu] = 2$ to the renormalized exact one, $[\mu_R] = d$, induces a change in all critical exponents corresponding to magnitudes measured away from the critical point with respect to their corresponding mean field values. Moreover, as happens in pair-coagulation, there is no further renormalization required for the fields nor the diffusion constant [14, 16, 23] and therefore all exponents can be exactly computed at any dimension. For instance, the scaling dimension of the field is Λ^d and, hence, scales as μ_R, implying $\beta = 1$ in any dimension. Using the same logic one obtains $\nu_{||} = 2$, $\nu_\perp = 1$ for the correlation time and correlation length exponents, while right at the critical point $z = 2$. Using standard scaling relations, the density of particles as a function of time decays in one dimension with an exponent $\theta = 1/2$, while in $d = 2$ a similar calculation leads to logarithmic corrections and, in particular, to a decay $\ln(t)/t$, while β remains equal to 1 [14].

In order to prove that the fixed point with $\mu_c = 0$ is not just *a solution* but also the *only one* one should consider all the possible diagrams (even if this can be done only in a symbolic form [24]), write down the 4 Z-functions, analogous to eq.(4) for the 4 vertex functions (2 proportional to μ and 2 to σ in their bare form). Doing this, it is straightforward to check that 3 different and independent flow equations are obtained. The fourth one is not independent owing to the usual duality symmetry [6] but this is not important for the argumentation here. As there are only 2 independent bare parameters, there is no way to find a fixed point for this set of 3 independent equations except for the trivial one $\mu_c = 0$, which simultaneously satisfies in a trivial way the first 2 equations, and leads back to the preceeding calculation, to the symmetry preserving Eq.(4), and to the same set of exponents.

Note that in models in the DP class, where the naïve power counting is different, with $d_c = 4$, only 2 independent parameters in the flow equations need to be fine tuned to zero. The third one (corresponding to the highest order noise coefficient) is irrelevant (flows to zero) already at mean field level and, therefore, does not require fine tuning to vanish asymptotically. Hence, contrarily to the previous case, a non-trivial solution, $\mu_c \neq 0$ exists leading to a DP fixed point.

As pointed out in [15], the reversible reaction studied here and $2A \leftrightarrow 0$ share the same type of critical behavior. Indeed, the Hamiltonian in this latter case is $(\pi^2 - 1)(\mu - \sigma\phi^2)$ where, as before, μ and σ are the creation and annihilation rate respectively. The zero-energy manifolds are: $\pi = \pm 1$ and $\phi = \pm\sqrt{\mu/\sigma}$ (fig. 1e). They define a quadrangular structure, as the one described above, but in this case $\phi = 0$ is not an invariant manifold: the vacuum state is accessible but it is not stable, so it is not properly an absorbing state phase transition. A perturbative analysis analogous to the one above can be done for the present case (indeed this is the model studied in [14, 15]) and leads to the same set of critical exponents; here the common factor $\pi^2 - 1$ plays the role of the subtle symmetry above.

Finally, for reversible coagulation reactions involving *triplets* instead of pairs, $A \leftrightarrow 3A$, we obtain similar results: vanishing critical point and exactly computable exponents, but the critical dimension is $d_c \leq 1$ in this case [1].

EXTENSION TO NON-REVERSIBLE REACTIONS

A careful but simple inspection of the arguments in the preceeding section leads to the conclusion that none of the reported results depends on the fact that the creation reaction is of the form $A \to 2A$. As will be argued in this section, most of them apply to more general *non-reversible* processes with creation reactions as $A \to nA$. For these, the creation part of the Hamiltonian is $\mu(\pi^n - \pi)\phi$, which together with the pair-coagulation terms $\sigma(\pi^2 - \pi)\phi^2$ guarantees that $(\pi^2 - \pi)$ can be extracted as a common factor for non-reversible bare Hamiltonians, and hence π is dimensionless and $\pi = 0$ and $\pi = 1$ are zero-energy solutions as in the $n = 2$ case. For example for $n = 3$, $H = \mu(\pi^3 - \pi)\phi - \sigma(\pi^2 - \pi)\phi^2 = (\pi^2 - \pi)[\mu(\pi + 1)\phi - \sigma\phi^2]$. The existence of such a common factor in the *bare* Hamiltonian is, as explained before, guaranteed if and only if the vacuum state is not accessible.

For the family of non-reversible models with $n > 2$, the geometry of the zero-energy manifolds of the bare Hamiltonian *is not a rectangular one* as occurs for the reversible

set of reactions with $n = 2$ [1]. For instance, for $n = 3$ one obtains a *trapezoidal geometry* (zero-energy solutions: $\pi = 0$, $\pi = 1$, $\phi = 0$ and $\phi = (\pi + 1)\mu/\sigma$, (see figure 1c), but the overall *topology* is not changed. Indeed, as the critical point is approached the difference between the rectangle and the trapezium becomes negligible, and at criticality this manifold merges with the $\phi = 0$ one. Analogously, for $n = 4$ one obtains a quadrangle with 3 straight lines and a curved one ($\phi = (\pi^2 + \pi + 1)\mu/\sigma$) (see fig. 1d), which also becomes closer and closer to the horizontal line upon approaching the critical point. In all cases, the non-trivial μ-dependent manifold merges with the absorbing-state one $\phi = 0$ at the critical point, and this constitutes the main trait of this class as will be illustrated here: in DP they intersect at criticality at a single point, in PC they intersect in one point in the active phase and merge at criticality [1], while in the class under scrutiny, they do not intersect in the active phase and merge at the critical point.

Note that, as π is dimensionless, all the different processes for different values of n are equally relevant at mean-field level (they just differ in powers of π). As a consequence, the naïve scaling dimensions for any $n > 2$ are as in the preceeding section, leading to $d_c = 2$. It is also important to realize that *higher-order processes generate effectively lower-order ones* (in particular, $A \to 2A$ is always generated) and all of them share the same degree of *naïve* relevancy. The generation of lower-order processes induces changes in the zero-energy manifolds, and leads to combinations of the previous "pure" topologies obtained for creation processes involving only one value of n. In order to render the theory renormalizable, lower order terms have to be included in the bare Hamiltonian, with coefficients proportional to μ (as they have to vanish as $\mu \to 0$) that we call μ_n. Indeed, from now on we study physical processes where various types of creation events with different values of n are simultaneously present (in particular $n = 2$ is always generated).

At a perturbative level, one can proceed as before, and separate corrections proportional and not proportional to μ. The first notorious difference with the reversible case is that upon renormalizing, the shape of some zero-energy manifolds is deformed if terms with $n \geq 3$ are present. Indeed, owing to the fact that the coefficients of π^n, with $n \geq 3$, in these generalized processes renormalize as

$$\mu'_{n,R} = \mu_n \left(1 - \frac{n(n-1)}{2}\sigma\Sigma\right) \tag{6}$$

up to one loop [25] while the corresponding "mass" coefficient renormalizes as in Eq.(4), different corrections are generated for these two coefficients equal at a bare level (therefore, the need to use different names, $\mu'_{n,R}$ and $\mu_{n,R} = \mu_R$, for the two of them, as a generalization of the single equation for μ_R in Eq.(4)). Eq.(6) shows that the scaling dimensions of the non-linear term coefficients, $\mu'_{n,R}$ varies with n: the lower the value of n, the more relevant the corresponding non-linear term.

Proceeding as before, it is straightforward to see by performing a perturbative expansion around $d_c = 2$ that the only way to find a solution of the RG flow equations at any arbitrary order in perturbation theory is by fixing $\mu = 0$. For instance, considering creation reactions with $n = 2$ and $n = 3$, one has 3 independent parameters: σ, μ_2 and μ_3 and 5 independent flow equations. Hence, at criticality all creation rates have to vanish, and one recovers the fixed point and exponents in the previous section, so *the universality class is preserved under the introduction of non-reversible reactions*. Note that, in order to extend the calculation in the preceeding section, it has been enough to impose that all creation terms are proportional to μ. This ensures that all of them vanish at the critical point and generate no extra diagrammatic correction, but they do not need to be all equal as happens in the reversible case.

We should also emphasize that, as said before, the mass-like terms associated to each n-creation process are all equally relevant and they all renormalize as μ_2, while the μ'_n renormalize differently for $n \geq 3$ (see Eq.(6)) and hence, $\pi^2 - \pi$ is not a common factor of the *renormalized* Hamiltonian, except at the critical point $\mu = 0$ where such a subtle symmetry is restored. The common factor or subtle symmetry invoked all along the calculation in the previous section, equivalent to the existence of reversibility or detailed balance, is not a necessary condition. As a consequence, the zero-energy manifold structure is affected: the topology shown in figures 1c and 1d is not stable under the RG flow, $\pi = 1$ is not a zero-energy manifold of the renormalized Hamiltonian, and the phase portrait structure becomes more complicated (see Fig. 1f).

Despite of this, we observe that from the phase-portrait point of view, *a key ingredient, not altered upon introducing non-reversible reactions, is the fact that the non-trivial μ-dependent manifold and the trivial one $\phi = 0$ do not intersect in the active phase and merge into a degenerate manifold at the critical point*. Therefore, the main ingredient of this universality class is *not* the reversibility nor the existence of a common factor in the renormalized Hamiltonian but the way in which the non-trivial manifold and the trivial one merge [26]. In summary, *reversibility is a sufficient but not a necessary requirement*.

For completeness' sake let us comment on another family of reactions without an accessible vacuum, including higher-order creation reactions as $kA \to (k + n)A$ with $k \geq 2$ which exhibit a different type of scaling behavior. These have to be complemented with higher order annihilation reactions as $jA \to lA$ with $j \geq k$ and $j > l \geq 1$ in order to ensure the existence of a bounded stationary state. For instance, taking $2A \to 3A$ (with rate μ) as a creation reaction together with $2A \to A$ (rate σ), we need another annihilation reaction, as $3A \to 2A$ (with

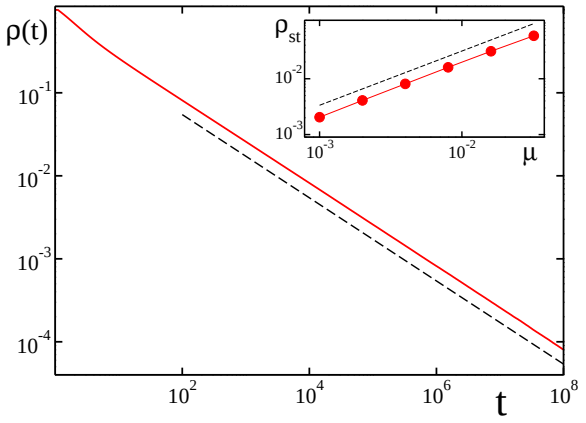


FIG. 2: (Color online) Results of Monte-Carlo simulations for $2A \to A$ and $A \to 3A$ implemented in a bosonic way in one dimension. The decay of the order parameter at criticality ($\mu_c = 0$) is given by $t^{-0.5}$ (main plot). This result is well known as at $\mu = 0$ this model coincides with pair-coagulation. The order-parameter critical exponent is perfectly fitted by $\beta = 1.00$ (inset). Very similar results are obtained for the reversible case, $n = 2$, as well as for higher-order non-reversible cases, as $n = 4$ and $n = 5$.

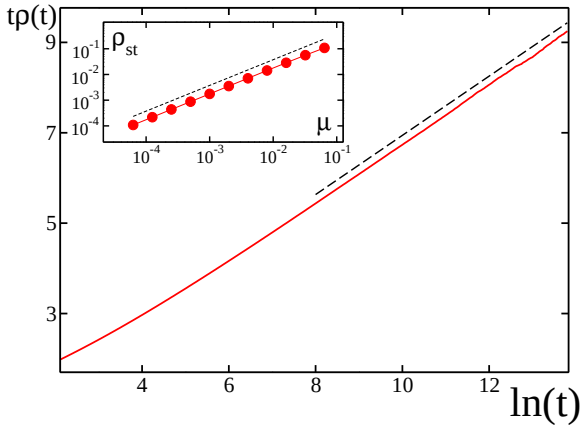


FIG. 3: (Color online) Results of Monte-Carlo simulations for $2A \to A$ and $A \to 3A$ implemented in a bosonic way in two dimensions. The decay of the order parameter at criticality ($\mu_c = 0$) is proportional to $\ln(t)/t$ (main plot). At its critical point $\mu = 0$, this model coincides with pair-coagulation, for which this is a well known results. The order-parameter critical exponent is perfectly fitted by $\beta = 1.00$ (inset).

rate $\lambda > \mu$) to have a well defined stationary state. For this case, even if $\pi = 0$ and $\pi = 1$ are constant energy solutions (at least at a bare level) the manifold $\phi = 0$ is degenerated, and the non-trivial zero-energy solution, $\phi = \mu/\lambda - \sigma/(\lambda\pi)$, intersects the line $\phi = 0$ at σ/μ and becomes singular at $\pi = 0$, originating a very different topology from the one above. This topology corresponds to the PCPD class [1, 9]. Therefore, *creation from pairs* in systems without an accessible vacuum leads to a different universality class.

Finally, for non-reversible *coagulation reactions involving triplets* ($3A \to A$ and $A \to nA$) we obtain again that the universality class remains unchanged with respect to the corresponding reversible reaction (see last paragraph of the previous section).

MONTE-CARLO SIMULATIONS

In order to verify the above field theoretical predictions we have performed Monte-Carlo simulations of the reversible reactions (reproducing some existing results [12]) and, more relevantly, *non-reversible* set of reactions: $2A \to A$ together with $A \to nA$ with $n = 3,\ 4,\ 5$. We have considered two different implementations: a *bosonic* one in which the number of particles at every site in a lattice is unrestricted (which is the one directly related to the bosonic field theory presented here), and a *fermionic* one with number occupancy restricted to be 0 or 1. For both of them the same type of numerical experiments have been performed. Figure 2 shows our main results for $n = 3$ in the bosonic implementation. In the main body, we plot the time evolution of the order-parameter as a function of time for a one-dimensional lattice of size 2^{20}. A clean power-law decay is observed at $\mu_c = 0$ with slope $\theta = 0.500(1)$ in a log-log plot. This is not surprising as at $\mu = 0$ this model coincides with pair-coagulation. Direct measurements of the order-parameter as a function of the distance to the critical point (upper inset) lead to $\beta = 1.00(1)$. Also, from measures of the mean-squared distance associated with two point correlation functions [15] one can easily measure $z = 2$ in all the cases under consideration. All the remaining exponents can be derived using standard scaling laws, providing a full check of the theoretical predictions for the bosonic model. For the fermionic model we obtain identical conclusions. In $d = 2$ mean-field exponents with logarithmic corrections have been measured confirming that $d_c = 2$ (see figure 3). In $d = 3$, Jack et al. [15] showed by means of Monte-Carlo simulations that the scaling is Gaussian as expected.

We have also verified that for the sets of non-reversible reactions with $n = 4$ and with 5 one obtains the same set of critical exponents, supporting again the theoretical conclusions.

SUMMARY

We have shown using field theoretical arguments and verified by means of Monte-Carlo simulations that all reaction-diffusion processes including pair coagulation $2A \to A$ and creation in the form $A \to nA$ belong to the same universality class, regardless of whether the reactions are reversible or not. The critical point is located at zero creation rate, and all critical exponents are

controlled by the well-known pair-coagulation renormalization group fixed point and can be exactly computed. These conclusions are in agreement with exactly known results for the reversible model in one dimension [12]. The main ingredient of this class of absorbing-state transitions is that the vacuum state is not accessible and creation occurs from individual particles. If creation occurs only from pairs then scaling is as in the PCPD class while, as soon as a reaction making the vacuum accessible, as, for example, $2A \to 0$ is switched-on, the system recovers standard DP scaling. There are also models in this universality class as $2A \leftrightarrow 0$ where the vacuum state is accessible but in these cases it is not stable: $0 \to 2A,\ 3A, ...$, so they are not properly absorbing-state transitions.

We have shown that the topology of the zero-energy manifolds is very important to unveil universality classes, but there could be many subtleties leading to surprises. We hope that this work fosters new studies to clarify some of the still-standing problems on universality in non-equilibrium critical phenomena.

– We thank A. Kamenev, I. Dornic, G. Ódor, F. de los Santos, and H. Chaté for useful comments, and H. Hinrichsen for a very helpful exchange of correspondence. We acknowledge financial support from the Spanish MEyC-FEDER, project FIS2005-00791 and from Junta de Andalucía as group FQM-165.

[1] V. Elgart and A. Kamenev, Phys. Rev. E **74**, 041101 (2006).

[2] J. Marro and R. Dickman, *Nonequilibrium Phase Transitions in Lattice Models* (Cambridge University Press, Cambridge, 1998).

[3] H. Hinrichsen, Adv. Phys. **49**, 815 (2000). G. Ódor, Rev. Mod. Phys. **76**, 663 (2004). G. Grinstein and M. A. Muñoz, in *Fourth Granada Lectures on Computational Physics* edited by P. L. Garrido and J. Marro, Lecture Notes in Physics Vol 493 (Springer, Berlin), p. 223.

[4] Additionally, they also identify some other classes (not enumerated here) marginal in one dimension.

[5] Some of these universality classes might have a better representation in terms of two species [9]. Also, some other classes with absorbing states as, for example, the one of stochastic sandpiles [R. Dickman, M. A. Muñoz, A. Vespignani, and S. Zapperi, Braz. J. of Physics **30**, 27 (2000)] usually represented by two density fields, admit a one-species non-Markovian description. Hence, saying that there are 5 non-trivial one-species absorbing-state universality classes is somehow ambiguous.

[6] J.L. Cardy and R.L. Sugar, J. Phys. A **13**, L423 (1980). H.K. Janssen, Z. Phys. B **42**, 151 (1981). P. Grassberger, Z. Phys. B **47**, 365 (1982).

[7] P. Grassberger, F. Krause, and T. von der Twer, J. Phys. A **17**, L105 (1984). See also [3] for reviews on this.

[8] O. Al Hammal, H. Chaté, I. Dornic, and M. A. Muñoz, Phys. Rev. Lett. **94**, 230601 (2005).

[9] For an overview, see M. Henkel and H. Hinrichsen, J. Phys. A **37**, R117 (2004).

[10] Indeed, the results in [1] can be taken as a new evidence supporting the conclusion that PCPD differs from DP.

[11] G. Ódor, Phys. Rev. E **73**, 047103 (2006).

[12] M. Burschka, C. R Doering, and D. ben-Avraham, Phys. Rev. Lett. **63**, 700 (1989). D. ben-Avraham, M. Burschka, and C. R Doering, J. Stat. Phys. **60**, 695 (1990). See also, K. Krebs, M. Pfannmueller, B. Wehefritz, and H. Hinrichsen J. Stat. Phys. **78**, 1429 (1995). H. Hinrichsen, S. Sandow, and I. Peschel, J. Phys. A. **29**, 2643 (1996). H. Hinrichsen, K. Krebs, and I. Peschel, Z. Phys. B **100**, 105 (1996). H. Hinrichsen, V. Rittenberg, and H. Simon, J. Stat. Phys. **86**, 1203 (1997).

[13] D. ben-Avraham, Mod. Phys. Lett. B **9**, 895 (1995). D. ben-Avraham, in *Statistical Mechanics in one-dimension*, Ed. V. Privman, pg. 29, Cambridge University Press, Cambridge 1997.

[14] J. Cardy and U. C. Taüber, Stat. Phys. **90**, 1 (1998).

[15] In R. Jack, P. Mayer, and P. Sollich, J. Stat. Mech. (2006) P03006, conclusions identical to ours are obtained for the *reversible case* by exploiting the existence of *detailed balance*.

[16] L. Peliti, J. Phys. A **19**, L365 (1986). M. Droz and L. Sasvari, Phys. Rev. E **48**, R2343 (1993). See also, B. P. Lee, J. Phys. A **27**, 2633 (1994). B. P. Lee and J. Cardy, J. Stat. Phys. **80**, 971 (1995). J. Cardy, *Field Theory and Nonequilibrium Statistical Mechanics*, (Troisième cycle de la Physique en Suisse Romande, 1998-1999, semestre d'été). http://www-thphys.physics.ox.ac.uk/users/JohnCardy/home.html

[17] For a recent and nice review see, U. C. Täuber, M. Howard, and B. P. Vollmayr-Lee, J. Phys. A. **38**, R79 (2005). For non-perturbative calculations see L. Canet, H. Chaté, and B. Delamotte, Phys. Rev. Lett. **92**, 255703 (2004). See also [14].

[18] M. Doi, J. Phys. A **9**, 1479 (1976). L. Peliti, J. Physique, **46**, 1469 (1985). P. Grassberger and M. Scheunert, Fortschr. Phys. 28, 547 (1980). C. J. DeDominicis, J. Physique **37**, 247 (1976). H. K. Janssen, Z. Phys. B **23**, 377 (1976). P. C. Martin, E. D. Siggia, and H. A. Rose, Phys. Rev. A **8**, 423 (1978).

[19] J. Zinn-Justin, *Quantum field theory and critical phenomena*, (Oxford University Press, 4th edition, 2002).

[20] Alternatively, Eq. (1) can also be obtained by employing the Poissonian transformation method [21]. It has been proved that both formalisms are completely equivalent: M. Droz and A. McKane, J. Phys A: Math. Gen. **27**, L467 (1994)).

[21] C. W. Gardiner, *Handbook of Stochastic Methods*, Springer-Verlag, Berlin and Heidelberg, 1985. M. A. Muñoz, Phys. Rev. E. **57**, 1377 (1998).

[22] By imposing the first derivatives of the Hamiltonian, H, with respect to ϕ and π to vanish, one can identify the homogeneous classical (mean-field) stationary solution. The non-trivial expectation value of ϕ is $\langle\phi\rangle = \mu/\sigma$ and hence, at mean-field level, the critical point is located at $\mu_c = 0$ and the order-parameter critical exponent is $\beta = 1$. At this critical point the coefficients of both the linear-deterministic and the leading noise terms vanish.

[23] As the diagrams in eq.(3) are marginally divergent in $d = 2$, their derivatives do not require extra renormalization, and therefore the fields do not have anomalous dimensions [16].

[24] Simple inspection of diagrams for different vertices, even

those having the same topology (for example, the one-loop diagrams contributing to DP [6]), readily leads to the conclusion that combinatorial factors are different, and therefore, each bare constant would have distinct corrections.

[25] More complicated two-loop diagrams are generated when higher order vertices are generated, breaking the super-renormalizability of the theory (see, for instance, fig. 3b in [14]).

[26] Analogously, non-reversibility can also be included in models without a proper absorbing state as $2A \leftrightarrow 0$ by switching on reactions as $0 \to 3A,\ 4A, ...$, without affecting their critical behavior.